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A Forecasting Model for Japan's Unemployment Rate

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Abstract

This note aims to achieve a parsimonious fractionally-integrated autoregressive and moving average (ARFIMA) model for recent time series data of Japan's unemployment rate. A brief review of the ARFIMA model is provided, leading to econometric modeling of the data in the ARFIMA framework. It is demonstrated that the preferred ARFIMA model is a satisfactory representation of the data and is useful as a forecasting device.

Keywords: *Unemployment Rate, Hysteresis, ARFIMA Model, Forecasting.*

JEL Classification Codes*: C53, J64*

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1. Introduction

This note presents a forecast-oriented model for Japan's unemployment rate. The model belongs to a class of fractionally-integrated autoregressive and moving average (ARFIMA) models. The introductory section briefly reviews the related literature.

It is known that shocks tend to have rather persistent effects on unemployment rates; this phenomenon is usually seen as evidence for hysteresis (see Blanchard and Summers, 1987). Thus, conventional time series models such as autoregressive models, which do not necessarily allow for such long-lasting effects, may face much difficulty in giving satisfactory descriptions of unemployment rates. With a view to accounting for persistency in unemployment rates, it would be preferable to use a class of long-memory or fractionally-integrated time series models. See Gil-Alana (2001), Caporale and Gil-Alana (2007), inter alia, for this line of research.

In order to estimate a parametric ARFIMA model, Sowell (1992) develops a maximum likelihood procedure. Doornik and Ooms (2003) refines the likelihood-based procedure, and Doornik and Ooms (2004) then applies the method to modelling inflation data in the UK and US. The empirical success achieved by Doornik and Ooms (2004) is remarkable in that the ARFIMA modelling allows us to obtain a parsimonious data-representation which can be used for out-of-sample forecasting.

This note estimates a parsimonious ARFIMA model for Japan's unemployment rate, demonstrating the model's adequacy and forecasting performance. The organisation of this note is as follows. Section 2 reviews an ARFIMA model and its time series properties. Section 3 then estimates an ARFIMA model describing Japan's unemployment rate in recent years. This section investigates the performance of the preferred model in terms of 1-step out-of-sample forecasting. Finally, Section 4 provides concluding remarks. All the numerical analyses and graphics in this paper use OxMetrics/PcGive (Doornik and Hendry, 2007).

2. Review of ARFIMA Model

According to Doornik and Ooms (2003, 2004), an ARFIMA model for a scalar time series u_t, which corresponds to Japan's unemployment rate in this note, is given by

$$\Phi(L)(1-L)^d (u_t - \mu_t) = \theta(L)\varepsilon_t, \quad \text{for } t = 1,\ldots,T, \tag{1}$$

where L denotes lag operator so that $\Phi(L)$ and $\Theta(L)$ are defined as

$\Phi(L) = (1-\phi_1 L - \ldots - \phi_p L^p)$ and $\Theta(L) = (1+\theta_1 L + \ldots + \theta_q L^q)$, and the time-varying mean of u_t is represented by μ_t, the error term ε_t has independent and identical normal $N(0, \sigma^2)$ distributions. The indices p and q are integers, while d is a real number such that $(1-L)^d$ represents fractional difference operator, defined by the binomial theorem as the following series:

$$(1-L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j .$$

The ARFIMA model above is denoted as ARFIMA(*p*, *d*, *q*). It is also assumed that the roots of $\Phi(L) = 0$ and $\Theta(L) = 0$ all lie outside the unit circle and do not have any common roots, so that the ARMA-part of the model above is invertible and stationary.

Let us define $z_t = u_t - \mu_t$, which is a zero-mean process integrated of order d, or I(*d*). Time series properties of z_t vary according to the parameter d (see Brockwell and Davis, 1993, Ch.13). The process z_t is covariance stationary if $d < 0.5$ and regarded as a long memory process if $0 < d < 0.5$, in which case its autocovariance function declines rather slowly. If $-0.5 < d < 0$, the process is referred to as intermediate memory. Note that, according to Odaki (1993), the condition of $d > -1$ is required for the process z_t being invertible.

The parameter d is, however, usually unknown hence it needs to be estimated from the data. Doornik and Ooms (2003, 2004) demonstrate that, provided $-1 < d < 0.5$, it is possible to estimate d using the method of maximum likelihood along the same line as Sowell (1992). Moreover, Doornik and Ooms (2003, 2004) show that, if z_t is a non-stationary process with $d > 0.5$, differencing or the method of nonlinear least squares (see Beran, 1995) should be employed to estimate d.

The next section, using the ARFIMA modelling methodology, pursues a forecasting model for Japan's unemployment rate.

3. Modelling and Forecasting Japan's Unemployment Rate

This section aims to attain a forecasting model for monthly data of Japan's unemployment rate, u_t. In order to achieve it in the ARFIMA framework above, it is necessary to choose such economic variables as may correspond to μ_t in Equation (1). One of the potentially important variables would be nominal wage inflation, according to macroeconomic models incorporating the Phillips curve (see

Romer, 2001, Ch.5, inter alia). Hence, an annual inflation measure based on the log of Japan's monthly earnings index of various lag orders, together with several autoregressive terms such as u_{t-1} and u_{t-2}, are included in the model in order to represent the time-varying mean of the unemployment rate. The data frequency is monthly, and the sample period for estimation runs from January 1995 to August 2008 (denoted as 1995.1-2008.8 hereafter), covering a recent period just before the outbreak of a global financial crisis, associated with the US housing price bubble, in September 2008. The data source is International Financial Statistics, published by International Monetary Fund.

Seasonality is observed in the data overview, thus the ARMA part of an initial unrestricted model is so constructed that it may capture the underlying seasonal pattern. The initial ARFIMA model allowing for seasonality is then estimated using the method of maximum likelihood, and insignificant regressors are eliminated from the model step by step. The process of the model reduction leads to a parsimonious ARFIMA model for Japan's unemployment rate as follows:

$$\left(1-\underset{[0.02]}{0.98}L^{12}\right)(1-L)^{\overset{-0.19}{[0.06]}}\left(u_t-\underset{[0.07]}{0.36}-\underset{[0.02]}{0.92}u_{t-1}+\underset{[1.04]}{5.06}\Delta_{12}w_{t-1}\right)=\left(1-\underset{[0.12]}{0.74}L^{12}\right)\varepsilon_t,$$
(2)

where $\Delta_{12}w_t$ is the 12th-order difference of the log of Japan's monthly earnings index and the figures in the square brackets are standard errors.

Figure 1 is provided for the purpose of graphic analysis of the parsimonious model. Figure 1 (a) plots actual and fitted values, denoted as u_t and $\hat{u}_t$, respectively. Figure 1 (b) shows scaled residuals of the model. Figure 1 (c) displays a density function for the residuals, together with a Normal density function, while Figure 1 (d) presents correlogram for the residuals. Figure 1 provides much evidence in support of the adequacy of the parsimonious model; the tracking of the data is remarkably good and the model's residuals appear to follow the Gaussian distribution with no serial correlation. It is therefore justifiable to conclude that Equation (2) is a data-congruent representation.

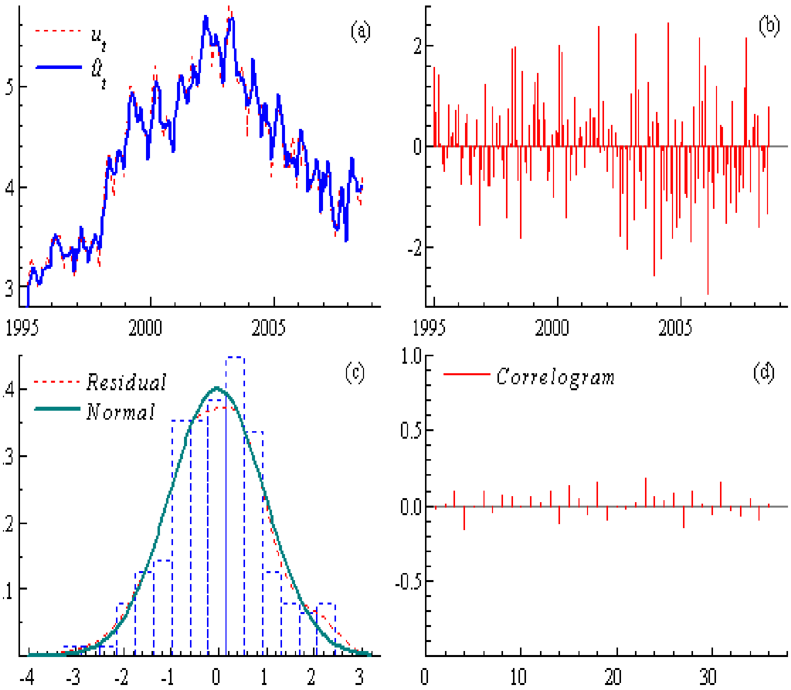

Figure 1: Graphic Analysis of the Parsimonious ARFIMA Model

In Equation (2), the demeaned process z_t is given by

$$z_t = u_t - 0.36 - 0.92u_{t-1} + 5.06\Delta_{12} w_{t-1}, \tag{3}$$

so that the fractional difference parameter, d, is defined with respect to z_t in Equation (3). According to (2), the estimate for d lies in the range of $-0.5 < d < 0$, suggesting that z_t is seen as an intermediate memory process.

Equation (2) is now treated as a benchmark model, paving the way for the investigation of the model's forecasting performance. Figure 2 records sequences of 1-step out-of-sample forecasts, according to four different forecasting horizons. Figure 2 (a) displays a sequence of 1-step forecasts over 2005.9-2006.8, using the ARFIMA model estimated from the data covering 1995.1-2005.8. Similarly, Figure 2 (b), (c) and (d) present: 1-step forecasts over 2006.9-2007.8 (1995.1-2006.8), 2007.9-2008.8 (1995.1-2007.8), and 2008.9-2009.6 (1995.1-2008.8), respectively. The figure in each parenthesis denotes a sample period for estimation, preceding each forecasting horizon. According to the figure, the overall forecasting performance is good; the actual values lie in the 95% confidence intervals (expressed as shaded fans) in most cases. Some influences of the global financial crisis seem to be reflected in a series of forecasts around 2008.9 and 2008.10, as shown in Figure 2 (d). The subsequent forecasts, however, come to perform well again, in spite of the fact that the level of the unemployment rate has risen dramatically. Figure 2 (d) suggests that the financial crisis has triggered off an upsurge in the level of the unemployment rate, but has less significant impacts on the model's forecasting performance itself. Overall, Figure 2 allows us to conclude that the ARFIMA model is regarded as a reliable forecasting device for Japan's unemployment rate.

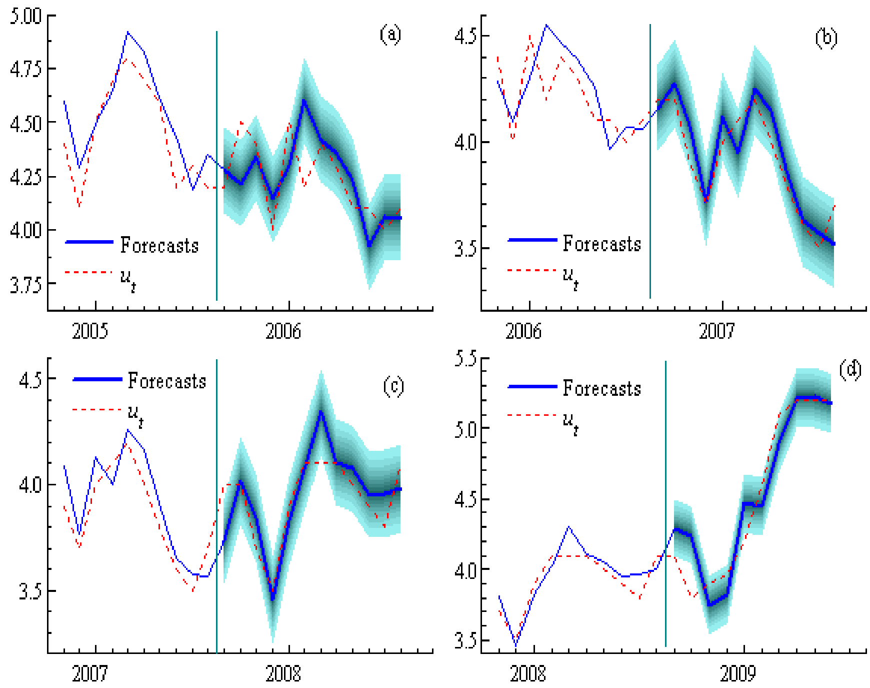

Figure 2: Sequences of 1-Step Forecasts of the Parsimonious ARFIMA Model

Next, we proceed to a comparative analysis of out-of-sample forecasts. A competing model for the ARFIMA model (2) is an autoregressive (AR) model of lag order 1 incorporating a drift term. Two sorts of measures for forecast accuracy are adopted: one is a root mean square error (RMSE), while the other is a mean absolute percentage error (MAPE). We may, in principle, consider that the smaller these statistics are, the better the corresponding forecasts in terms of accuracy. These statistics are calculated using 1-step forecasts and actual values for the four types of forecast horizons, in accord with Figure 2, and are presented in Table 1 for each model. The table shows that, in most cases, both of these measures for the ARFIMA model are much smaller than those for the AR model. Hence, the overall evidence in Table 1 is in support of Equation (2), giving weight to the validity of the ARFIMA model as a reliable forecasting device, although we may require more statistical evidence in order to arrive at a decisive conclusion in the comparative study.

Table 1: Measures of Forecast Accuracy: ARFIMA and AR Models

<RMSE>

	2005.9-2006.8	2006.9-2007.8	2007.9-2008.8	2008.9-2009.6
ARFIMA	0.176	0.110	0.131	0.205
AR	0.239	0.191	0.205	0.240

<MAPE>

	2005.9-2006.8	2006.9-2007.8	2007.9-2008.8	2008.9-2009.6
ARFIMA	3.238	2.385	2.590	3.785
AR	4.540	4.416	4.394	3.877

4. Concluding Remarks

This note has attained a parsimonious ARFIMA model for recent time series data of Japan's unemployment rate. It is demonstrated that the preferred model is a satisfactory representation of the data and is of much use for forecasting purposes.

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